Mobius Madness & Klein Bottle Lesson Plan

Amount of time: 5-10 mins.
Try this at home!

Materials
- Paper of various colors
- Many pairs of scissors
- Many tape dispensers
- Pencils
- Box for scrap paper/recycling
- Klein Bottle
- Optional: water
- Optional: paper towels

Set-up Instructions
1. Neatly lay out some paper, scissors, tape, pencils, and a box for recycling. Set the Klein Bottle on the table (away from the edge!!)

SAFETY!
- Klein Bottles are made of glass and should be handled with care. If something is broken, close down the demo, clear people out of the area, and carefully clean up any broken glass.
- Make sure that students are responsible with scissors.

Lesson’s Big Idea
- High School: Mobius Strips are 3D objects that are one continuous side (as opposed to a regular loop of paper, which has an inner surface and an outer one). A Klein Bottle is a 4D object that is all one surface (its’ inside is its’ outside!). Of course we don’t actually have 4-dimensional objects, so the glass bottle is just a model of what it would look like.
- Middle School: Mobius Strips are all one side, not an inner half and an outer part like a normal paper loop. Klein Bottles are related to Mobius Strips -- they are all one surface, so its insides are its outsides.

Background Information
• **Topology** is a branch of **geometry**. It involves surfaces and bending shapes. In topology, you're allowed to make any change to a shape that can be undone. From Wayne State: “A circle is the same as a triangle or a square, because you just pull on parts of the circle to make corners and then straighten the sides, to change a circle into a square. Then you just `smooth it out' to turn it back into a circle.” Another example: A bowl can be flattened out into a plate.

• **Mobius Strips** are objects studied in topology. They are related to cylinders, but have one continuous side instead of an inside and an outside. They also have a boundary that you can identify and trace.

• When you slice a Mobius Strip, some crazy geometry and math happens. Depending on the number of twists in the loop and the number of cuts, you can end up with a longer loop, more twists, or several loops joined together in a knot (see Wolfram MathWorld in the references).

• **Klein Bottles** are related to Mobius loops. If you could stitch two Mobius strips together along their edge, you would get a Klein Bottle. It would have one surface, inside the same as outside, and no ‘boundary’ like the paper loop has an edge. However, it would also be in 4 dimensions, which we cannot actually create with the current laws of physics.

**Instructional Procedure**

1. Invite each student to grab a piece of paper, scissors, and tape. They should each cut two long strips from their piece of paper, about 1.5 inches wide. Have them tape all the way around the joint for one long strip. Do not tape them into a closed loop yet!

2. While they are doing this, make a regular loop of paper. Ask them how many ‘sides’ it has and invite them to trace their finger around the inside
and outside of the loop to demonstrate that it has two sides.

3. Take the long loop of paper, twist one end, then tape it into a closed loop. Have the students do the same thing. It should end up looking like this:

![Image of a twisted loop of paper](image.png)

4. Remind them of the standard loop, which had two sides. Encourage them now to take a pencil now and trace the Mobius loop, starting on the ‘outside’ and continuing for ... as long as it takes. They should discover that their loop has only one side! That is what makes a Mobius loop cool -- it looks like it has two sides, but it's actually all one continuous side.

5. Go back to your regular loop and cut it in half, all the way around. You should end up with two distinct loops (ask the students what they think will happen, just for good measure). Now for the fun part! Invite the students to cut along their Mobius strip, all the way around. Ask them what they think will happen. If all has gone well, the students should each have a single loop still! It will be longer, and will contain an extra twist.

![Image of two Mobius loops](image.png)

6. If students want to stop at this surprising point, they can. Otherwise, invite them to do the same thing again -- cut the long Mobius loop all the way around, and ask what they think will happen. They will find themselves now with two Mobius loops, linked in the middle, with no way of getting them apart!
7. If students want to continue slicing the loops, they can: from this point onward, they will simply be creating more and more loops, all connected at a tangled knot in the middle.

**Assessment/Sample questions you can ask**
- How many sides does a Mobius Strip have?
- What happens when you stitch two Mobius Strips together on their edge?
- Why can’t we actually have Klein Bottles?

**Clean Up**
- Gather all the scissors and tape neatly into a bin. If there are half-sheets of scrap paper, feel free to keep them for future use. Recycle scraps. Gently wrap the Klein Bottle up and put it back in the bin.

**References**
- Wayne State University: [http://www.math.wayne.edu/~rrb/topology.html](http://www.math.wayne.edu/~rrb/topology.html)
- Wolfram MathWorld: [http://mathworld.wolfram.com/MoebiusStrip.html](http://mathworld.wolfram.com/MoebiusStrip.html)
- Acme Klein Bottles: [http://www.kleinbottle.com/whats_a_klein_bottle.htm](http://www.kleinbottle.com/whats_a_klein_bottle.htm)

**Mathematics Common Core Standards**
- High School Geometry
  - G-GMD (relationships between 2- and 3-D objects)
  - G-MG (modeling with geometry)
- Also fits with most grade-level science and engineering practices in the Next Generation Science Standards (experiments, noticing patterns, hypotheses, etc.)