Tower of Hanoi Lesson

Plan

Amount of time demo takes: Up to 20 mins.

Try this at home!

Materials

- Tower of Hanoi set
- Rules sheet
- Whiteboard and marker (optional)

Set-up Instructions

1. Get out the kit(s) and make sure they are presented in the correct order (biggest disk on the bottom, moving up to the smallest disk). Note: the more disks there are, the harder this puzzle is to solve!

2. If your audience is younger (or time is limited), use fewer disks. The math of this will be explained in the Background Info section, but here is a guide:
   - a. 3 disks 7 moves to solve
   - b. 4 disks 15 moves to solve
   - c. 5 disks 31 moves to solve
   - d. 6 disks 63 moves to solve
   - e. 7 disks 127 moves to solve
   - f. 8 disks 255 moves to solve
   - g. 9 disks 511 moves to solve

3. Optional: Set up a whiteboard to keep track of how many moves it takes to solve the puzzle (or make it a timed competition!).

SAFETY: Safe demo

Lesson's Big Idea

- The Tower of Hanoi is a mathematical puzzle and brain teaser. You have to apply logical thinking to move the entire stack of discs within the rules (see below) and try to do so in as few moves as possible.

Background Information

- This puzzle was invented by French mathematician Edouard Lucas in 1883, inspired by legend. There are many similar legends surrounding the origins of this puzzle: priests are charged with moving an entire stack of rock disks
(up to 64 of them) from one location to another, following several simple rules. In some of these legends, the world will end (or the priests will reach Nirvana) when the puzzle has been solved.

- Many computer science courses use this puzzle as a programming exercise. For our purposes, it’s just a great brain teaser. Here is a diagram illustrating how to solve a 4-disk puzzle (from Wolfram MathWorld, read down the left column, then the right):

- At the end of the day, you can easily find out the number of moves it will take to solve a 3-peg Tower of Hanoi puzzle using this simple equation:

\[ h_n = 2^n - 1 \]

where \( h_n \) is the number of moves it will take and \( n \) is the number of disks. Math fact: numbers that look like \( 2^n - 1 \) are called Mersenne Numbers.

### Instructional Procedure

1. Invite students to tackle a cool challenge. Explain the rules of the puzzle to them (see included rule cards):
   
   The goal is to move the entire stack from one peg to another.
   
   a. You can move only one disk at a time.
   
   b. No disk can be placed on top of a smaller disk.
   
   c. A disk can move either onto an empty peg OR onto a larger disk.
   
   d. In the end, the stack must look exactly like the starting stack.

2. Give the students some time to begin tackling the puzzle. They can keep
track of their number of moves as an added challenge! If they seem stuck, you can begin explaining some solution processes using the diagrams included in the kit.

3. If students do solve the puzzle (and want to have friendly competition), feel free to write their first name and number of moves on the whiteboard. Keep a running tally through the day, or you can go for simple speed rather than number of moves.

**Assessment**

1. How could you make this puzzle at home?
2. How hard was it to solve the puzzle?
3. What rules could you change to make the puzzle easier? Harder?

**Clean Up**

- Disassemble the tower kit and store it neatly back in its box. Clean off the whiteboard and pack away all materials.

**References**


**Common Core Mathematics Standards**

- 8th Grade Math 8.EE.A.1: Integer exponents
- High School Mathematics
  - HSA-SSE.A.1: Interpret expressions representing a quantity
  - HSA.REI.B.3: Solving equations ... in one variable
Tower of Hanoi Rules

The goal is to move the entire stack from one peg to another. When you are done, the final stack must look exactly like the starting stack. There are only two rules:

You can move only one disk at a time.

You may never place a disk on top of a disk that is smaller than it.

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